SOME ECONOMIC EFFECTS OF IMMIGRATION: A GENERAL EQUILIBRIUM ANALYSIS

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INTRODUCTION [1/]

Immigration, in general, has many economic consequences for the receiving country, and attempts to analyse them abound in the literature. Most frequently examined in the literature are the effects on: the excess aggregate demand for domestic goods ([16] and [18]); the balance of payments ([18]); per capita income ([21] and [23]); the distribution of the national product ([21] and [23]); the stock of human capital (the so called "brain drain" controversy) ([1], [15], and [26]); labour shortages (the advantages and disadvantages of meeting labour shortages by immigrant labour) ([22]). In this paper we concentrate on two aspects of the impact of immigration on the full-employment path of the economy of the host country: the effects of immigration on the real wage and per capital real income of the indigenous population and on the distribution of the national product.

Under the generally valid assumption that immigrants bring with them less capital then is owned by the average indigenous resident, the qualitative results appear to be straightforward. As Mishan and Needleman state in [21], "... if one supposes that an immigrant increase of the population is a microcosm of the indigenous population, save in respect of ownership of capital, the qualitative results are obvious enough: an exogenous fall in the capital-labour ratio tends to have a regressive effect on the distribution of income, and, unless economics of scale are

sufficiently strong, tends to reduce wages and per capita real income."[2/] A good portion of this paper is concerned with dropping the assumption that the immigrant population "is a microcosm of the indigenous population." We show that when it is deleted the results cease to be so straightforward.

For our analysis we adopt a general equilibrium framework similar to that employed by Diewert [11] in his analysis of the effects of unionization on wages and employment. The comparative statics technique which we consequently adopt has the advantage over the simulation technique used by Mishan and Needleman in their paper that it allows one, rather easily, to solve algebraically for the immigrant induced changes in the economy as functions of various basic parameters, such as elasticities of substitution in production and consumption, and immigrant characteristics. We work out expressions for the induced changes in wage rates, real wages, per capita real income and occasionally, for labour's share of the national income.

The paper may be outlined as follows: In section I. we assume a one industry, two factor model, the factors called labour and capital. Allowing for a complete general equilibrium in the output market and in the two factor markets, assuming fixed factor supplies, and making the assumption referred to above, we derive the "obvious" results.

The more interesting results follow when we further disaggregate the economy. In section II. we assume a three factor model, capital and two types of labour. Once again, allowing for a complete general equilibrium in all markets, and assuming fixed factor supplies, we work out the induced changes in both wage rates and in the real wages of both types of workers.

sufficiently strong, tends to reduce wages and per capita real income.[10] A good portion of this paper is concerned with dropping the assumption that the immigrant population "is a microcosm of the indigenous population," to show that when it is dropped the results cease to be so strikingly adverse.

For our analysis we adopt a general equilibrium framework similar to that employed by Johnson [11] in his analysis of the effects of unionization on wages and employment. The comparative statics technique which we consequently adopt has the advantage over the simulation technique used by Berry and Soligo in their paper that it allows one, rather easily, to solve algebraically for the immigrant induced changes in the economy as functions of various basic parameters, such as elasticities of substitution in production and consumption, and immigrant characteristics. We work out expressions for the induced changes in wage rates, rent rates, per capita real income and occasionally, for labour's share of the national income.

The paper may be outlined as follows. In section I, we assume a one industry, two factor model, the factors called labour and capital. Allowing for a complete general equilibrium in the output market and in the two factor markets, assuming fixed factor supplies, and making the assumption referred to above, we derive the "adverse" results.

The more interesting results follow when we further disaggregate the economy. In section II, we assume a three factor model, capital and two types of labour. Once again, allowing for a complete general equilibrium in all markets, and assuming fixed factor supplies, we work out the induced changes in both wage rates and in the real wages of both types of workers.

Finally, in III., we divide the economy into two production sectors, reverting to two factors of production. Immigrant workers are not assumed to be dispersed between the sectors precisely as the indigenous workers. Neither do we assume that they have identical preferences for the two consumption goods. The labour supply decision is endogenized and we allow for labour and capital mobility between sectors. We show that some very "unexpected" results are possible.

The many simplifying assumptions we have made will become clearer to the reader as he studies the models in each section. We note that many of them, such as the small numbers of goods and factors, the inelastic supply of capital, capital mobility between sectors, and the absence of foreign trade and the balance of payments problem, may be relaxed, but at the cost of introducing additional parameters into the model. (The increasing complexity of the models in sections I. through III. illustrates the costs involved in introducing additional complications).

In order to illustrate the formulae which we develop, we present some numerical examples in each section. They relate to the Canadian economy and immigration into Canada in the 1969-70 time period.[3/] The reader is warned, however, about drawing any conclusions on the basis of the numbers we generate. Any conclusions depend upon the assumptions of our models, which may not be even approximately valid in reality.

I.

Mimicing Diewert's description, we may describe our economy as follows: We assume that it consists of N consumer-workers, each wishing to maximize his consumption of good y. Each consumer-worker is capable of supplying capital services K and labour services L. By a convenient choice of units for y, K and L, we may assume that, during the time period under consideration, each consumer worker consumes one unit of y and supplies one unit of K and L. Also, we assume that all consumer-workers and producers behave competitively, i.e., as price takers.

We further assume that the technology involved in producing y can be represented by means of a two factor constant returns to scale production function,[4/] with the usual regularity properties. By the Shephard Duality Theorem [10], technology may be represented equally well by means of the unit cost function $C(r,w)$, where $C(r,w) \equiv$ minimum cost of producing one unit of y given wage rate w and rental rate on capital r.

Now by Shephard's Lemma [24], the firm's cost minimizing demand for a factor of production can be obtained by partially differentiating the firm's total cost function with respect to that factor's price. Thus the initial demand for labour can be obtained as

$$\frac{\partial C(r,w)}{\partial w} \cdot y \equiv C_w(r,w) \cdot y = C_w(r,w) \cdot N \quad ,$$

where the last equality follows from the fact that each consumer demands one unit of y and there are N consumers in our economy. Upon equating initial factor demands to initial factor supplies, we obtain the following system of equations:

I.1 $$\text{(demand for labour)} \quad C_w(r,w) \cdot N = N \quad \text{(supply)}$$

$$\text{(demand for capital)} \quad C_r(r,w) \cdot N = N \quad \text{(supply).}$$

Since we are in a general equilibrium context, only relative prices matter and thus we are allowed one price normalization. We set $r \equiv 1$. (Note that under this normalization $w = \frac{wN}{rN}$, the economy's labour-capital ratio). Using the normalization $r = 1$, setting $\bar{w}$ equal to the economy's initial labour-capital ratio and using equations I.1, we find that the first order partial derivatives of the cost function are given by:

I.2 $$C_w(1,\bar{w}) = C_r(1,\bar{w}) = 1.$$

In the ensuing discussion, we will want to express the second order partial derivatives of the unit cost functions in terms of elasticities of substitution. Let us shift for a moment from a unit cost function defined over two factor prices to a total cost function defined over output and N factor prices, i.e., consider $C(y;p_1,\ldots,p_N)$, the minimum cost of producing output y, given input prices $p_1,\ldots,p_N$.[5/] Then if the cost function is twice differentiable with respect to input prices, the <u>partial elasticity of substitution</u> (Allen [ ], Uzawa [ ]) between inputs i and j can be defined as:

I.3 $$\sigma_{ij}(y;p_1,\ldots,p_N) \equiv \frac{C(y;p_1,\ldots,p_N)\, C_{ij}(y;p_1,\ldots,p_N)}{C_i(y;p_1,\ldots,p_N)\, C_j(y;p_1,\ldots,p_N)}, \quad i \neq j$$

(Of course $C_i(y;p_1,\ldots,p_N)$, $C_j(y;p_1,\ldots,p_N)$ and $C_{ij}(y;p_1,\ldots,p_N)$ denote the partial derivatives $\frac{\partial C}{\partial p_i}(y;p_1,\ldots,p_N)$, $\frac{\partial C}{\partial p_j}(y;p_1,\ldots,p_N)$ and $\frac{\partial^2 C}{\partial p_i \partial p_j}(y;p_1,\ldots,p_N)$ respectively).

Equation I.3 may be used to express C_{ij} in terms of σ_{ij} (and C_i, C_j, C) for $i \neq j$. For $i=j$, we may use the following formula:[6/]

I.4
$$C_{ii}(y; p_1, \ldots, p_N) = - \sum_{\substack{k=1 \\ k \neq i}}^{N} p_k \, C_{ik} / p_i \, .$$

Returning to our two factor model, let σ be the initial elasticity of substitution between labour and capital. Using the appropriate version of I.3 and I.4 along with I.2, we obtain the following expressions for the second order partial derivatives of our unit cost function:

I.5
$$C_{rw}(1,\bar{w}) = \frac{\sigma}{1+\bar{w}} \, , \quad C_{ww}(1,\bar{w}) = \frac{-\sigma}{\bar{w}(1+\bar{w})} \, , \text{ and } C_{rr}(1,\bar{w}) = \frac{-\bar{w}\,\sigma}{1+\bar{w}} \, .$$

We now disturb the initial equilibrium defined by equations I.1, by introducing an additional I consumer-workers into the economy.[7/] They are each capable of supplying "a" units of labour and "k" units of capital services during the time period under consideration. In general, a new equilibrium (relative) wage w will be established as a result of this influx. If we denote by $D(w)$ the total demand for consumption of good y, at wage rate w, then the new equilibrium position is described by:

I.6
$$\text{(demand for labour)} \quad C_w(1,w) \cdot D(w) = N + aI \text{ (supply)}$$
$$\text{(demand for capital)} \quad C_r(1,w) \cdot D(w) = N + kI \text{ (supply)}.$$

This is a system of 2 simultaneous equations in one unknown, w. However, by Walras' Law (ie., because of the fact that consumer-workers spend all of their labour and capital income on y) the equations are dependent and equivalent to the single equation:

I.7
$$\frac{C_w(1,w)}{C_r(1,w)} = \frac{N + aI}{N + kI} \, .$$

Our goal is to determine the change in the wage rate w which has been induced by immigration. Our basic strategy is to totally differentiate equation I.7 with respect to w and I, and then to evaluate the partial derivative $\frac{\partial w}{\partial I}$ at the initial equilibrium position where $w = \bar{w}$ and $I = 0$. Upon doing this and using I.5, we obtain the following:

I.8 $$\frac{dw}{\bar{w}} = \frac{dI}{N}\left(\frac{k-a}{\sigma}\right).$$ [8/]

Equation I.8 describes the percent change in the wage rate due to a (small relative) influx of immigrants. It is the "obvious" result to which we have referred in the introduction. Wages will increase (relative to rentals) only if immigrants supply more capital than labour in the units which have been defined; that is, wages will increase or decrease according as the capital-labour ratio of immigrants is greater than or less than the ratio of the indigenous population. The usual case where $k-a < 0$, will result in $dw < 0$.[9/] The larger the elasticity of substitution, the smaller is this regressive effect.[10/]

We now turn to the induced change in the real income of an indigenous worker. We assume that the preferences of each indigenous consumer-worker can be expressed by a utility function $U = U(y)$, where U is a non-negative, nondecreasing, linear homogeneous function.

Under our competitive assumptions, the price of a good is equal to its unit cost, so that we have:

I.9 price of one unit of $y \equiv p = C(1,w)$.

At the initial equilibrium position, $w = \bar{w}$, and

$$C(1,\bar{w}) = C_r(1,\bar{w}) + wC_w(1,\bar{w}) = 1+\bar{w}.$$

Therefore, the initial price, $\bar{p}$, is given by:

I.10 $$\bar{p} = 1+\bar{w}.$$

Now, by appropriately defining the units in which we measure utility, we may assume that the indirect utility function, $u(w)$, which expresses the consumer-worker's utility level as a function of wages, is simply equal to the number of units of y which the consumer-worker can afford to buy at wage rate w; that is, we have:

.11 $$u(w) = \text{income} \Big/ \text{price of } y = \frac{1+w}{C(1,w)} .$$

Totally differentiating I.11 and evaluating the derivative at the initial position, we find that $\frac{\partial u}{\partial I} = 0$. This is consistent with the fact that at the initial equilibrium position, $I = 0$, and welfare is at a globel maximum. Thus calculus techniques will not help us to determine the change in real per capita income.[11/]

If we consider a worker who has no capital services to offer, then his total income is equal to w and so his indirect utility function is given by:

[.12 $$u(w) = \frac{w}{C(1,w)} .$$

We find that $\frac{du}{\bar{u}} = \frac{dw}{\bar{w}} \cdot \frac{1}{1+\bar{w}}$, and so his real wage will also fall if his "money" wage falls.[12/]

Lastly, we consider labour's share of the national income. Let us define $S(I)$ by:

I.13 $$S(I) = \frac{(N+aI)\ w}{(N+aI)\ w + (N+kI)}$$

Then, if $\bar{S} \equiv \frac{N\bar{w}}{N(1+\bar{w})} = \frac{\bar{w}}{1+\bar{w}}$, labour's initial share, we find that:

I.14 $$\frac{dS}{\bar{S}} = \frac{dI}{N}\left(\frac{k-a}{\sigma}\right)\frac{(1-\sigma)}{1+\bar{w}} .$$

Even if immigration has a regressive effect, labour's share of the national income will increase if $\sigma > 1$.

Numerical Example

We take the initial equilibrium position to be that which approximates the position of the Canadian economy during the 1969 calendar year. This equilibrium is then perturbed by immigration during 1970.

We take N to be the average size of the Canadian labour force during 1969, approximately 8,162,000 people ([8]), and I to be the total number of immigrants in 1970 destined to the labour force, about 77,723, as indicated in Immigration Statistics 1970 [5]. Since we are using labour force populations, we can, by assuming the same distribution of hours worked for the indigenous and immigrant working forces, set a = 1.

Estimates of capital resources pose a greater problem. We use the Hood and Scott [13] estimate of the capital-output ratio for Canada for the year 1955, which was 2.4. Therefore, using a G.N.P. figure of 78,560 million dollars for 1969, we compute capital stock per labour force member to be 2.31×10^4 . Immigrants in 1970 declared total savings of about 304 million dollars upon assuming immigrants status, so that we (very crudely) estimate capital holdings per immigrant labour force member by 3.9×10^3.[13/] Thus, $k = \dfrac{3.9 \times 10^3}{2.31 \times 10^4} = .17$.

Using I.6 therefore, we find that:

I.15 $$\frac{dw}{\bar{w}} = \frac{-dI}{N}\left(\frac{.83}{\sigma}\right)$$

Thus, our simple model and the crude numerical estimates we have made, indicate that immigration results in a downward pressure on relative wage rates. Assuming in turn that $\sigma = \frac{1}{2}$, 1, and $1\frac{1}{2}$, and using the fact that $\frac{dI}{N} = .95\%$, we find that

$$\frac{dw}{\bar{w}} = \left\{ -1.6\%,\ -.79\%,\ -.53\%. \right.$$

The induced percent change in the real wage of a consumer-worker having no income from capital may now be calculated using equation I.12. We estimate $\bar{w}$, the ratio of labour income to capital income in Canada in 1969, to be 4.0.[14/] Thus, the changes in the real wage corresponding to the above changes in money wages are given by:

$$\frac{du}{\bar{u}} = \left\{ -.32\%,\ -.16\%,\ -.11\%. \right.$$

II.

The model in I. assumed that immigrant and native workers provide the same "type" of labour; i.e., that the immigrant and indigenous labour forces have identical occupational and skill compositions. This is generally not the case, and so we now alter our model so as to reflect the effect of the different compositions of the two labour forces.

We assume a three factor model, capital K, and two types of labour L_1 and L_2. The economy consists of N consumer-workers, each wishing to maximize his consumption of good y. Each consumer-worker is capable of providing 1 unit of capital services K. However, αN of them (type 1 workers) supply 1 unit of L_1 per period, and $(1-\alpha)N$ of them (type 2 workers) supply 1 unit of L_2 per period, where $0 \le \alpha \le 1$. The units of measurement for K, L_1, and L_2 are thus defined implicitly. The unit of measurement for y is defined by assuming that the total consumption of y, during the period under consideration, is equal to N. Also, all consumer-workers and producers are price takers.

The technology involved in producing y is represented by means of a three factor constant returns to scale production function, with the usual regularity properties.[15/] Applying the Shephard Duality theorem we derive the unit cost function $C(r, w_1, w_2)$, where w_1 and w_2 are the wage rates for type 1 and type 2 workers respectively, and r is the rental rate on capital. Using Shephard's Lemma as in the first section, we may describe the economy's initial equilibrium position by the following equations:

II.1

(demand for L_1)	$C_{w_1}(r,w_1,w_2) \cdot N = \alpha N$	(supply)
(demand for L_2)	$C_{w_2}(r,w_1,w_2) \cdot N = (1-\alpha)N$	(supply)
(demand for capital)	$C_r(r,w_1,w_2) \cdot N = N$	(supply).

Making the normalization $r = 1$, and setting $\bar{w}_1$ and $\bar{w}_2$ equal to the initial relative wage rates, we deduce that the first order partial derivatives of the cost function are given by:

II.2 $$C_{w_1}(1,\bar{w}_1,\bar{w}_2) = \alpha \quad , \quad C_{w_2}(1,\bar{w}_1,\bar{w}_2) = 1-\alpha \quad , \quad C_r(1,\bar{w}_1,\bar{w}_2) = 1.$$

In order to find the second order partials of the cost function we use II.2 and the appropriate versions of I.3 and I.4. Letting σ_i be the initial elasticity of substitution between capital and L_i, $i = 1,2$, and letting σ_{12} denote the initial elasticity of substitution between the two types of labour, we derive the following expressions for the second order partials:

II.3 $$C_{w_1w_2}(1,\bar{w}_1,\bar{w}_2) = \frac{\alpha(1-\alpha)\sigma_{12}}{1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2} \qquad C_{w_1r}(1,\bar{w}_1,\bar{w}_2) = \frac{\alpha\sigma_1}{1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2}$$

$$C_{w_2r}(1,\bar{w}_1,\bar{w}_2) = \frac{(1-\alpha)\sigma_2}{1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2} \qquad C_{w_1w_1}(1,\bar{w}_1,\bar{w}_2) = \frac{\alpha[(1-\alpha)\bar{w}_2\sigma_{12}+\sigma_1]}{w_1[1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2]}$$

$$C_{w_2w_2}(1,\bar{w}_1,\bar{w}_2) = -\frac{(1-\alpha)[\alpha\bar{w}_1\sigma_{12}+\sigma_2]}{w_2[1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2]} \qquad C_{rr}(1,\bar{w}_1,\bar{w}_2) = -\frac{[\alpha\bar{w}_1\sigma_1+(1-\alpha)\bar{w}_2\sigma_2]}{1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2}.$$

The initial equilibrium defined by equations II.1 is disturbed by the influx of an additional I consumer-workers into the economy, among them aI type 1 workers and $(1-a)I$ type 2 workers. In all, they supply kI units of capital services. The new equilibrium equations are:

II.4
$$\begin{array}{lll} \text{(demand for } L_1\text{)} & C_{w_1}(1,w_1,w_2)\cdot D(w_1,w_2) = \alpha N+aI & \text{(supply)} \\ \text{(demand for } L_2\text{)} & C_{w_2}(1,w_1,w_2)\cdot D(w_1,w_2) = (1-\alpha)N+(1-a)I & \text{(supply} \\ \text{(demand for } K\text{)} & C_r(1,w_1,w_2)\cdot D(w_1,w_2) = N+kI & \text{(supply),} \end{array}$$

where $D(w_1,w_2)$ is the aggregate demand for the consumption of y when wage rates w_1 and w_2 prevail. By Walras' Law these equations are dependent and so are equivalent to the following system:

II.5
$$\frac{C_{w_1}(1,w_1,w_2)}{C_{w_2}(1,w_1,w_2)} = \frac{\alpha N + aI}{(1-\alpha)N+(1-a)I},$$

$$\frac{C_{w_1}(1,w_1,w_2)}{C_r(1,w_1,w_2)} = \frac{\alpha N + aI}{N+kI}.$$

In order to determine the changes in w_1 and w_2 induced by immigration, we totally differentiate equations II.5 with respect to w_1, w_2 and I, and calculate the partial derivatives $\frac{\partial w_1}{\partial I}$ and $\frac{\partial w_2}{\partial I}$ evaluated at the initial equilibrium situation. Using equations II.2 and II.3, we obtain the following pair of equations:

I.6
$$\frac{dw_1}{\bar{w}_1}(1-\alpha)\left[-\sigma_{12}(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)-\sigma_1\right] + \frac{dw_2}{\bar{w}_2}(1-\alpha)\left[\sigma_{12}(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)+\sigma_2\right] =$$
$$\frac{dI}{N}\frac{(a-\alpha)}{\alpha}\left[1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2\right]$$

$$\frac{dw_1}{\bar{w}_1}\alpha\left[-(1-\alpha)\bar{w}_2\sigma_{12}-\sigma_1(1+\alpha\bar{w}_1)\right] + \frac{dw_2}{\bar{w}_2}\alpha\left[(1-\alpha)\bar{w}_2(\sigma_{12}-\sigma_2)\right] =$$
$$\frac{dI}{N}\left(\frac{a}{\alpha}-k\right)\alpha\left[1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2\right]$$

Denote by $|A|$ the matrix of coefficients of this system. The determinant of A, $|A|$ **is given by**

$$|A| = [\sigma_1\sigma_2+\sigma_{12}(\alpha\bar{w}_1\sigma_1+(1-\alpha)\bar{w}_2\sigma_2)](1+\alpha\bar{w}_1+(1-\alpha)w_2)\alpha(1-\alpha).$$

Assuming that $|A| \neq 0$, we solve the system and arrive at the following:

II.7
$$\frac{dw_1}{\bar{w}_1} = \frac{dI}{N}\frac{\sigma_2+\sigma_{12}(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)}{\sigma_1\sigma_2+\sigma_{12}(\alpha\bar{w}_1\sigma_1+(1-\alpha)\bar{w}_2\sigma_2)}\left[\left(k+\frac{a}{\alpha}\right) - \frac{(a-\alpha)}{\alpha(1-\alpha)}\frac{(1-\alpha)\bar{w}_2(\sigma_2-\sigma_{12})}{\sigma_2+\sigma_{12}(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)}\right]$$

$$\frac{dw_2}{\bar{w}_2} = \frac{dI}{N}\frac{\sigma_1+\sigma_{12}(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2}{\sigma_1\sigma_2+\sigma_{12}(\alpha\bar{w}_1\sigma_1+(1-\alpha)\bar{w}_2\sigma_2)}\left[\left(k-\frac{1-a}{1-\alpha}\right) + \frac{(a-\alpha)}{\alpha(1-\alpha)}\frac{\alpha\bar{w}_1(\sigma_1-\sigma_{12})}{\sigma_1+\sigma_{12}(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)}\right].$$

The change in w_1 is due to a change in the capital-type 1 labour ratio, captured by the $(k-\frac{a}{\alpha})$ term, and a change in the type 1 labour-type 2 labour ratio, captured by the term $\frac{(a-\alpha)}{\alpha(1-\alpha)} \frac{(1-\alpha)\bar{w}_2(\sigma_2-\sigma_{12})}{\sigma_2+\sigma_{12}(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)}$.

The direction of the effect of these terms on the wage rate is determined by the sign of the factor F, $F = \frac{\sigma_2+\sigma_{12}(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)}{\sigma_1\ \sigma_2+\sigma_{12}(\alpha\bar{w}_1\sigma_1+(1-\alpha)\bar{w}_2\sigma_2)}$.

The straight forward results of section I. do not extend to our new model. For example, if F is positive (which is the case if all of the elasticities of substitution are non-negative, and at most one is equal to zero), we could have the following: w_1 will be induced to fall if immigrants bring with them proportionately little capital, i.e. $k-\frac{a}{\alpha} < 0$. On the other hand, a disproportionately large immigrant supply of type 2 labour, i.e., $a < \alpha$, will induce w_1 to rise if

$\frac{\sigma_2-\sigma_{12}}{\sigma_2+\sigma_{12}(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)} > 0$. The direction of the net change in w_1 will depend upon the relative intensities of these two effects, with an increase in w_1 quite possible. Indeed, if we also have $\frac{\sigma_1-\sigma_{12}}{\sigma_1+\sigma_{12}(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)} < 0$,

it is quite conceivable that w_2 could also rise, and this even though type 2 labour is proportionately the most abundant resource among immigrants. Of course, corresponding statements may be made with w_1 replaced by w_2.

Suppose that F is negative. (It is not at all inconsistent with empirical studies that exactly one of the σ_i, $i = 1,2$, be negative. For example, when type 1 and type 2 workers correspond to "blue collar" and "white collar" workers respectively, Berndt and Christensen [3] have

presented some evidence for this. In [4] they have proven that if σ_1, say, is positive, and σ_2 negative, and if the two types of labour are not perfect substitutes, then they may not be aggregated consistently into an aggregate labour input L. The possibility of such an aggregation was a basic implicit assumption made in I. Therefore the "surprising" result described below is surprising only if we do not keep this in mind). Then it is possible that both wage rates may rise, even if the immigrant labour force is identical to the indigenous work force, in the sense that $a = \alpha$, and even though $k < 1$.

The change in the average wage w_{av}, $w_{av} \equiv \alpha w_1 + (1-\alpha) w_2$, is given by:

I.8
$$\frac{dw_{av}}{\bar{w}_{av}} = \frac{\frac{dI}{N}}{\left[\sigma_1\sigma_2 + \sigma_{12}(\alpha\bar{w}_1\sigma_1 + (1-\alpha)\bar{w}_2\sigma_2)\right]} \left\{ \alpha\bar{w}_1\left(k - \frac{a}{\alpha}\right)\frac{[\sigma_2 + \sigma_{12}(\alpha\bar{w}_1 + (1-\alpha)\bar{w}_2)]}{\alpha\bar{w}_1 + (1-\alpha)\bar{w}_2} + (1-\alpha)\bar{w}_2\left(k - \frac{1-a}{1-\alpha}\right)\frac{[\sigma_1 + \sigma_{12}(\alpha\bar{w}_1 + (1-\alpha)\bar{w}_2)]}{\alpha\bar{w}_1 + (1-\alpha)\bar{w}_2} + \frac{(a-\alpha)}{\alpha(1-\alpha)}\frac{\alpha\bar{w}_1(1-\alpha)\bar{w}_2[\sigma_1 - \sigma_2]}{\alpha\bar{w}_1 + (1-\alpha)\bar{w}_2} \right\},$$

Compare for a moment equations I.8 and II.8. We have shown that because of the distinct compositions of the immigrant and native work forces, the model in this section indicates that the average wage may rise even if immigrants posess relatively little capital. Disregarding these distinct compositions, the models in I. and II. still yield qualitatively different results, because of aggregation errors in I.

Turning to the real income of each type of indigenous consumer-worker, we may proceed as in I. The indirect utility function of each type of worker, assumed to possess one unit of capital services, is given by:

II.9 $$u_i = \frac{1+w_i}{C(1,w_1,w_2)}, \quad i = 1,2.$$

We find that:

II.10 $$\frac{du_1}{\bar{u}_1} = \left[(1+\bar{w}_2)dw_1 - (1+\bar{w}_1)d\bar{w}_2\right]\frac{(1-\alpha)}{(1+\bar{w}_1)(1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)}$$

$$\frac{du_2}{\bar{u}_2} = \frac{-\alpha}{1-\alpha}\,\frac{1+\bar{w}_1}{1+\bar{w}_2}\,\frac{du_1}{\bar{u}_1}$$

Hence the real income of a type 1 or type 2 worker may rise even though his wage may fall. However, if the real income of a type 1 worker, say increases, the type 2 worker experiences a fall in his real income. The two changes moreover are such that the average real income, $\alpha u_1+(1-\alpha)u_2$, does not change, i.e., $d(\alpha u_1+(1-\alpha)u_2) = 0$, consistent with the economy being initially at a global welfare maximum.

As in I. it is now possible to work out the change in real wages and in each factor's share of the national product.

Numerical Example

As in the previous numerical example, we consider the 1969-70 period. Type 2 workers are those in the managerial or professional and technical occupation groups, as defined in the Labour Force Survey. The remainder of the labour force constitute workers of type 1. We call type 2 and type 1 workers "professionals" and "nonprofessionals" respectively.

Labour Force Survey data indicate that in 1969 about 23% of employed workers were "professionals". Since unemployment was lower among "professional" workers than among "nonprofessionals", we estimate that $\alpha = .80$, $1-\alpha = .20$.[16/]

In [6] we find the average earnings, by occupation, of the 1967 labour force. Though in this study earnings are defined to include somewhat more than labour income, we use these figures as a basis for estimating the distribution of labour income between the two types of workers. Assuming an approximately uniform change in wage rates for all occupations from 1967 to 1969, we estimate that:

$$\frac{\alpha\bar{w}_1}{\alpha\bar{w}_1 + (1-\alpha)\bar{w}_2} = .65 \quad , \quad \frac{(1-\alpha)\bar{w}_2}{\alpha\bar{w}_1 + (1-\alpha)\bar{w}_2} = .35$$

Now $\alpha\bar{w}_1+(1-\alpha)\bar{w}_2$ is the ratio of labour income to capital income. Based on data from [7] we estimate that $\alpha\bar{w}_1+(1-\alpha)\bar{w}_2 = 4.$ 17/ We deduce that $\bar{w}_1 = 3.25$ and $\bar{w}_2 = 7.0$.

The statistics on the intended occupations of immigrants in 1970, as reported in [5], indicate that about 67% of immigrant workers were "nonprofessionals" and 33% "professionals". Therefore, we have that

$$\frac{a}{\alpha} = \frac{.67}{.80} = .84 \quad , \quad \frac{1-a}{1-\alpha} = \frac{.33}{.20} = 1.65, \text{ and so } \frac{a-\alpha}{\alpha(1-\alpha)} = -.81.$$

Immigrants provide proportionately more "professional" labour than "nonprofessional" labour.

As in Section I., we estimate that $k = .17$. Consequently, using equations II.7 and II.8 we find:

$$\frac{dw_1}{\bar{w}_1} = \frac{dI}{N} \frac{\sigma_2+4\sigma_{12}}{\sigma_1\sigma_2 + \sigma_{12}(2.6\sigma_1 + 1.4\sigma_2)} \left[-.67 + 1.13 \frac{(\sigma_2 - \sigma_{12})}{\sigma_2 + 4\sigma_{12}} \right],$$

$$\frac{dw_2}{\bar{w}_2} = \frac{dI}{N} \frac{\sigma_1 + 4\sigma_{12}}{\sigma_1\sigma_2 + \sigma_{12}(2.6\sigma_1 + 1.4\sigma_2)} \left[-1.48 - \frac{2.1\,(\sigma_1 - \sigma_{12})}{\sigma_1 + 4\sigma_{12}} \right],$$

and

$$\frac{dw_{av}}{\bar{w}_{av}} = \frac{dI}{N} \frac{\left[.3\sigma_2 - 1.25\sigma_1 - 3.8\sigma_{12}\right]}{\sigma_1\sigma_2 + \sigma_{12}(2.6\sigma_1 + 1.4\sigma_1)}$$

In Table II.1 we calculate the induced changes in wages and real incomes for each type of worker, under alternative hypotheses concerning the magnitudes of the elasticities. (We assume that $\frac{dI}{N} = .95\%$.) A study by Berndt and Christensen [4] is the source of our hypotheses. Though their estimates are for the manufacturing sector in the United States only, for which they found time average values of about 8, 4 and -4 for σ_{12}, σ_1 and σ_2 respectively), we feel that it is not unreasonable to expect that the elasticities of the aggregate Canadian Production function lie somewhere in the range defined by the values used in the table.

The figures in Table II.1 show that in all cases wages are depressed, with the effect on type 2 wages being larger, because of the relative abundance, among immigrant labour, of "professional labour". (If we had grouped "managers" with type 1 workers, the difference between the effects on w_1 and w_2 would have been even more pronounced.) The figures for $\frac{dw_{av}}{\bar{w}_{av}}$ indicate, on the whole, a larger drop then indicated by the model in I. This may be explained by the fact that all combinations of elasticities satisfy $\sigma_1 > \sigma_2$. Thus, since $\frac{(a-\alpha)}{\alpha(1-\alpha)}$ is negative, so is the product $\frac{(a-\alpha)}{\alpha(1-\alpha)}(\sigma_1-\sigma_2)$, and so the relatively nonuniform distribution

Table II.1

THE INDUCED CHANGES IN THE WAGES AND REAL INCOMES OF "PROFESSIONALS" AND "NONPROFESSIONALS"

σ_{12} *	σ_1 **	σ_2 ***	$\frac{dw_1}{\bar{w}_1}$	$\frac{dw_2}{\bar{w}_2}$	$\frac{du_1}{\bar{u}_1}$	$\frac{du_2}{d\bar{u}_2}$	$\frac{dw_{av}}{\bar{w}_{av}}$
6	3	-2	- .94%	- 1.48%	+ .18%	- .38%	- 1.13%
6	3	-3	- 1.82%	- 2.55%	+ .26%	- .55%	- 2.07%
6	3	-4	-19.5 %	-26.7 %	+2.70%	-5.72%	-11.0 %
6	4	-2	- .60%	- .93%	+ .11%	- .23%	- .72%
6	4	-3	- .91%	- 1.41%	+ .17%	- .36%	- 1.08%
6	4	-4	- 1.83%	- 2.76%	+ .32%	- .68%	- 2.16%
6	5	-2	- .44%	- .76%	+ .10%	- .21%	- .56%
6	5	-3	- .61%	- 1.03%	+ .14%	- .29%	- .76%
6	5	-4	- .96%	- 1.60%	+ .22%	- .47%	- 1.18%
8	3	-2	- .88%	- 1.15%	+ .10%	- .21%	- .97%
8	3	-3	- 1.53%	- 1.99%	+ .18%	- .38%	- 1.69%
8	3	-4	- 5.5 %	- 7.01%	+ .13%	- .28%	- 6.03%
8	4	-2	- .57%	- .81%	+ .09%	- .19%	- .65%
8	4	-3	- .81%	- 1.14%	+ .12%	- .25%	- .93%
8	4	-4	- 1.37%	- 1.91%	+ .19%	- .40%	- 1.56%
8	5	-2	- .42%	- .64%	+ .08%	- .17%	- .49%
8	5	-3	- .55%	- .83%	+ .09%	- .19%	- .65%
8	5	-4	- .78%	- 1.17%	+ .14%	- .29%	- .92%

* σ_{12} is the elasticity of substitution between "professional" and "nonprofessional" labour.

** σ_1 is the elasticity of substitution between capital and "nonprofessional" labour.

*** σ_2 is the elasticity of substitution between capital and "professional" labour.

of immigrant labour has an added depressant effect on the average wage (see equation II.8).

For the elasticity combinations (6,3,-4) and (8,3,-4), the expression $\sigma_1\sigma_2+\sigma_{12}(\alpha\bar{w}_1\sigma_1+(1-\alpha)\bar{w}_2\sigma_2)$ assumes a very small positive value (see equations I.7), and as a result very large wage decreases are predicted. We note that for the elasticity values 5,3 and -4 for σ_{12}, σ_1, and σ_2 respectively, the expression assumes a small negative value, and large wage <u>increases</u> are predicted. It is clear that more precise knowledge about the elasticities of substitution is required before we can be certain of the magnitude, and perhaps even the direction, of the effect on wages of immigration.

Turning to real income, we see that the real income of "nonprofessionals" rises, while "professionals" experience a drop in real income. Of course, we have here assumed that all workers possess equal shares of capital. Under the more realistic assumption that more capital is concentrated among the generally wealthier "professionals", some combinations of elasticities will yield a decline in real income for "nonprofessionals" and consequently a rise in the real income of "professionals". Thus, information on the distribution of capital holdings is required before we can determine even the qualitative effect of immigration on the real incomes of each group of workers.

III.

The models in I. and II. assumed that the economy produced a single aggregate good, thereby assuming implicitly that immigrant and native consumer-workers have identical consumption preferences. In order to allow for distinct consumption preferences we now consider a two sector model, each sector producing a consumption good. The differences between the immigrant and native work forces are further taken account of in as much as we do not assume that the two work forces are dispersed identically between the sectors.

The analysis in this section differs further from that in the previous two sections in that the supply of labour is no longer assumed to be fixed. In an attempt to maximize his utility, the consumer-worker takes into account prevailing prices and wage rates before deciding upon the amount of labour that he will supply to each sector.

Initially our economy has N consumer-workers in it, each holding one unit of capital services and one unit of labour services which he supplies to producers during the period under consideration. The units of measurement for y_1 and y_2 are defined by assuming that, initially, each consumer-worker is consuming one unit of y_1 and one unit of y_2 per period. Also, we assume that all consumer-workers and producers are price takers.

Let α be the proportion of labour initially employed in sector 1 (so that the amount of labour initially employed in sector 1 is αN), and let β be the proportion of capital initially employed in sector 1.

The technology involved in producing y_1 and y_2 is represented by two factor constant returns to scale production functions

with the usual regularity properties. We have unit cost functions $C^1(r,w_1)$ and $C^2(r,w_2)$, where r is the rental rate on capital and w_1 and w_2 are the wage rates for labour in sectors 1 and 2 respectively.

Upon applying Shephard's Lemma and equating initial factor demands to initial factor supplies, normalizing by setting $r \equiv 1$, and defining the initial (relative) wage rates $\bar{w}_1$ and $\bar{w}_2$, we derive the following values for the first order partial derivatives of the unit cost functions:

III.1
$$C^1_w(1,\bar{w}_1) = \alpha \qquad\qquad C^1_r(1,\bar{w}_1) = \beta$$
$$C^2_w(1,\bar{w}_2) = 1-\alpha \qquad\qquad C^2_w(1,\bar{w}_2) = 1-\beta.$$

Under our competitive assumptions, the price of a good is equal to its unit cost, so that we have:

III.2
$$\text{Price of } y_1 \equiv p_1 = C^1(1,\bar{w}_1) \overset{\text{(initially)}}{=} \alpha\bar{w}_1+\beta$$
$$\text{Price of } y_2 \equiv p_2 = C^2(1,\bar{w}_2) \overset{\text{(initially)}}{=} (1-\alpha)\bar{w}_2 + (1-\beta).$$

The N consumer-workers in the economy are assumed to have identical preferences for the four "goods": The two production goods y_1 and y_2, and the two leisure goods Le_1 and Le_2. (Of course, the amount of leisure of each type consumed by the consumer-workers, is related to the labour supplied to each sector L_i, $i = 1,2$, by the equations $Le_i + L_i = H_i$, $i = 1,2$, where H_i is the maximum amount of labour which the consumer-worker is able to supply to the ith sector in the period under consideration). We assume that their preferences can be represented by a utility function $U(y_1,y_2,Le_1,Le_2)$, where U is a non-negative, nondecreasing, concave linear homogeneous function,[18/] and that they attempt to maximize U subject to their time and budget constraints.[19/] The counterpart to the unit cost function in the

context of consumer theory is the consumer's unit expenditure function, $e(p_1,p_2,w_1,w_2)$, which is defined to be the minimum expenditure necessary, at prices p_1, p_2, w_1 and w_2 for the "goods" y_1, y_2, Le_1, Le_2 respectively, in order that a utility level of at least 1 be attained, i.e., in order that $U(y_1,y_2,Le_1,Le_2) \geq 1$.

The indirect utility function $u(w_1,w_2)$ may be derived from the unit expenditure function as follows: since the consumer-worker spends his entire income on consumption of the "goods", we have that:

III.3 $$\text{(expenditure)} \quad u \cdot e(p_1,p_2,w_1,w_2) = 1 + w_1H_1 + w_2H_2 \text{ (income)}.$$

Rearranging and substituting III.2, we find that:

III.4 $$u(w_1,w_2) = \frac{1+w_1H_1 + w_2H_2}{e(C^1(1,w_1),\ C^2(1,w_2),\ w_1,w_2)} \quad .$$

By defining appropriately the units in which utility is measured, we may choose the following normalization:

III.5 $$e(C^1(1,\bar{w}_1),C^2(1,\bar{w}_2),\bar{w}_1,\bar{w}_2) = 1+\bar{w}_1H_1+\bar{w}_2H_2.$$

Therefore, the initial level of utility $\bar{u}$ is equal to one.

By applying Shephard's Lemma, we may determine each consumer-worker's initial demand for y_1 as $\frac{\partial e}{\partial p_1}(p_1,p_2,w_1,w_2) \cdot \bar{u} \equiv e_1(p_1,p_2,w_1,w_2) \cdot 1 = 1$, where the last equality follows from our assumption that initially each consumer is consuming one unit of y_1. We deduce that $e_1(\bar{p}_1,\bar{p}_2,\bar{w}_1,\bar{w}_2) = 1$. Proceeding similarly to consider the initial demands for the remaining three "goods", we derive the following initial values for the first order partial derivatives of the unit expenditure function:

III.6 $$e_1(\bar{p}_1,\bar{p}_2,\bar{w}_1,\bar{w}_2) = 1 \qquad e_2(\bar{p}_1,\bar{p}_2,\bar{w}_1,\bar{w}_2) = 1$$

$$e_3(\bar{p}_1,\bar{p}_2,\bar{w}_1,\bar{w}_2) = H_1-\alpha \qquad e_4(\bar{p}_1,\bar{p}_2,\bar{w}_1,\bar{w}_2) = H_2-(1-\alpha).$$

Denote by σ_i , i = 1,2, the elasticity of substitution between capital and labour in the ith sector, and by τ_{ij}, i,j = 1,2,3,4, the elasticity of substitution in consumption between the ith and jth "goods". The initial second order partial derivatives of both the unit cost functions and the unit expenditure function may now be expressed in terms of these elasticities and the remaining parameters, by using equations III.1, III.2, III.5 and III.6, along with the appropriate versions of equations I.3 and I.4. (In equations I.3 and I.4 replacing the cost function by the unit expenditure function, and the elasticities σ_{ij} by the elasticities τ_{ij} , yields the appropriate version for the consumer side.) We shall not calculate these second order partial derivatives here, for most of the equations to follow will not be written in terms of the various elasticities of substitution. Rather, in order not to complicate further an already complicated set of equations, the equations will be expressed in terms of the second order partials $\bar{C}^1_{ij}, \bar{C}^2_{ij}$, and $\bar{e}_{ij}$, where we have adopted the obvious abbreviation in our notation. Keeping in mind that these second order partials are directly proportional to the appropriate elasticities, we may interpret the equations as we did in I. and II.

We now disturb the initial equilibrium by allowing an additional I consumer-workers to immigrate into the country. They each have a linear homogeneous utility function $V(y_1,y_2)$, and a corresponding unit expenditure function $f(p_1,p_2)$.[20/] Upon entry into the country, they supply kI units of capital services, aI units of labour services to sector 1, and (1-a)I units of labour services to sector 2, per period.

If we denote by $D^i(w_1,w_2)$ the aggregate demand for the ith production good, i = 1,2, then we may apply Shephard's Lemma to deduce the following:

III.7 $$D^i(w_1,w_2) = e_i(C^1(1,w_1),C^2(1,w_2),w_1,w_2)\cdot\frac{N(1+w_1H_1+w_2H_2)}{e(C^1(1,w_1),C^2(1,w_2),w_1,w_2)} + f_i(C^1(1,w_1),C^2(1,w_2),w_1,w_2)\cdot\frac{I(k+aw_1+(1-a)w_2)}{f(C^1(1,w_1),C^2(1,w_2))},$$

$$i = 1,2.$$

Similarly, if we let $D_{Le_1}(w_1,w_2)$ and $D_{Le_2}(w_1,w_2)$ denote the aggregate demand by native workers for Le_1 and Le_2 respectively, we have:

III.8 $$D_{Le_i}(w_1,w_2) = e_{i+2}(C^1(1,w_1),C^2(1,w_2),w_1,w_2)\cdot\frac{N(I+w_1H_1+w_2H_2}{e(C^1(1,w_1),C^2(1,w_2),w_1,w_2)}$$

$$i = 1,2.$$

Lastly, let Q_1 and Q_2 be the amounts of goods 1 and 2 that would be demanded by immigrants at the initially prevailing wages and prices. We define ΔQ_1 and ΔQ_2 as follows:

III.9 $$Q_i = \frac{k+a\bar{w}_1+(1-a)\bar{w}_2}{1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2} + \frac{\Delta Q_i}{1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2}, \qquad i = 1,2.$$

Since each indigenous consumer-worker initially consumes one unit of each good, the term $\frac{k+a\bar{w}_1+(1-a)\bar{w}_2}{1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2}$ represents the amount of each good that the immigrant consumer-worker would consume, at the initial wages and prices, if they had the same preferences. Thus ΔQ_i is a measure of the difference between the consumption patterns of the immigrant and indigenous workers. Note that because immigrants spend all their income on consumption, we have that $\bar{C}^1\cdot\Delta Q_1+\bar{C}^2\cdot\Delta Q_2 = 0$.

We may now write the new factor equilibrium equations determined by our model. They are:

III.10 (demand for labour in sector 1)

$$C_w^1(1,w_1) \cdot D^1(w_1,w_2) = NH_1 - D_{Le_1}(w_1,w_2) + aI \qquad \text{(supply)}$$

(demand for labour in sector 2)

$$C_w^2(1,w_2) \cdot D^2(w_1,w_2) = NH_2 - D_{Le_2}(w_1,w_2) + (1-a)I \qquad \text{(supply)}$$

(demand for capital)

$$C_r^1(1,w_1) \cdot D^1(w_1,w_2) + C_r^2(1,w_2) \cdot D^2(w_1,w_2) = N + kI \quad \text{(supply).}$$

By Walras' Law these equations are dependent and so any two of them, say the first two, define a pair of simultaneous equations in the two unknowns w_1 and w_2. As in the preceding sections, we totally differentiate the first two equations in III.10 with respect to w_1, w_2 and I, and evaluate at the initial conditions. After making use of equations III.1 and III.2, and equations III.5 through III.9, we arrive at the following pair of equations:

III.11

$$dw_1[\alpha^2\bar{e}_{11}+2\alpha\bar{e}_{13}+\bar{e}_{33}+\bar{C}_{ww}^1] + dw_2[\alpha(1-\alpha)\bar{e}_{12}+\alpha\bar{e}_{14}+(1-\alpha)\bar{e}_{32}+\bar{e}_{34}]$$
$$= \frac{dI}{N}(a-\alpha Q_1)$$

$$dw_1[\alpha(1-\alpha)\bar{e}_{12}+\alpha\bar{e}_{14}+(1-\alpha)\bar{e}_{32}+\bar{e}_{34}] + dw_2[(1-\alpha)^2\bar{e}_{22}+2(1-\alpha)\bar{e}_{24}+\bar{e}_{44}+\bar{C}_{ww}^2]$$
$$= \frac{dI}{N}((1-a)-(1-\alpha)Q_2).$$

Denote by $|A|$ the determinant of the matrix of coefficients of this system. Note that $|A| > 0$ if $0 < \alpha < 1$, $0 < \beta < 1$ and at least two of the elasticities of substitution σ_1,σ_2 and τ_{ij}, $i,j = 1,2,3,4$, $i \neq j$, are positive, while the others are non-negative. We assume that $|A| > 0$, (though of course the equations may be solved if $|A| < 0$ and the discussion to follow altered accordingly), and derive the following solutions:

I.12

$$\frac{dw_1}{\bar{w}_1} = \frac{\frac{dI}{N}\left\{A_1(k-1) + A_2(a-\alpha) + A_3\Delta O_1\right\}}{\bar{w}_1(1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)\cdot|A|}, \text{ where}$$

$$A_1 = -\alpha\bar{C}^2_{ww} + \bar{e}_{12}\left[\alpha(1-\alpha)^2\frac{(1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)}{\bar{C}^2}\right] + \bar{e}_{23}\left[(1-\alpha)^2\frac{(\bar{C}^2+\alpha\bar{w}_1)}{\bar{C}^2}\right]$$

$$+ \bar{e}_{24}\left[\frac{\alpha(1-\beta)^2}{\bar{w}_2\bar{C}^2}\right] + \bar{e}_{14}\left[\frac{\alpha(\bar{C}^1+(1-\alpha)\bar{w}_2)}{\bar{w}_2}\right] + \bar{e}_{34}\left[\frac{\alpha\bar{w}_1+(1-\alpha)\bar{w}_2}{\bar{w}_2}\right],$$

$$A_2 = (1+\bar{w}_2)\bar{C}^2_{ww} + \bar{e}_{12}\left[-(1-\alpha)\frac{(1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)(\beta-\alpha)}{\bar{C}^2}\right]$$

$$+ \bar{e}_{23}\left[\frac{(1-\alpha)}{\bar{C}^2}\left[-(1-\alpha)(\bar{w}_1-\bar{w}_2) + (1-\beta)(1+\bar{w}_1)\right]\right] + \bar{e}_{24}\left[\frac{-(1+\bar{w}_2)(1-\beta)^2}{\bar{w}_2\bar{C}^2}\right]$$

$$+ \bar{e}_{14}\left[-\frac{\left[\alpha(\bar{w}_1-\bar{w}_2) + \beta(1+\bar{w}_2)\right]}{\bar{w}_2}\right] + \bar{e}_{34}\left[\frac{-(\bar{w}_1-\bar{w}_2)}{\bar{w}_2}\right],$$

$$A_3 = -\alpha\bar{C}^2_{ww} + \bar{e}_{23}\left[\frac{-\beta(1-\alpha)^2}{\bar{C}^2}\right] + \bar{e}_{24}\left[\frac{\alpha(1-\beta)^2}{\bar{w}_2\bar{C}^2}\right] + \bar{e}_{14}\left[\frac{\alpha(1-\beta)C^1}{\bar{w}_2\bar{C}^2}\right]$$

$$+ \bar{e}_{34}\left[\frac{\beta(1-\beta)}{\bar{w}_2\bar{C}^2}\left(\frac{\alpha\bar{w}_1}{\beta} - \frac{(1-\alpha)\bar{w}_2}{1-\beta}\right)\right].$$

$$\frac{dw_2}{\bar{w}_2} = \frac{\frac{dI}{N}\left\{B_1(k-1) + B_2(a-\alpha) + B_3\,\Delta O_2\right\}}{\bar{w}_2(1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)\cdot|A|}, \text{ where}$$

$$B_1 = -(1-\alpha)\bar{C}^1_{ww} + \bar{e}_{12}\left[\alpha^2(1-\alpha)\frac{(1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)}{\bar{C}^1}\right] + \bar{e}_{14}\left[\alpha^2\frac{(\bar{C}^1+(1-\alpha)\bar{w}_2)}{\bar{C}^1}\right]$$

$$+ \bar{e}_{13}\left[\frac{(1-\alpha)\beta^2}{\bar{w}_1\bar{C}^1}\right] + \bar{e}_{23}\left[(1-\alpha)\frac{(\bar{C}^2+\alpha\bar{w}_1)}{\bar{w}_1}\right] + \bar{e}_{34}\left[\frac{\alpha\bar{w}_1+(1-\alpha)\bar{w}_2}{\bar{w}_1}\right],$$

$$B_2 = -(1+\bar{w}_1)C^1_{ww} + \bar{e}_{12}\left[-\alpha\frac{(1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)(\beta-\alpha)}{\bar{C}^1}\right]$$

$$+ \bar{e}_{14}\left[\frac{-\alpha}{\bar{C}^1}[\alpha(\bar{w}_1-\bar{w}_2) + \beta(1+\bar{w}_1)]\right] + \bar{e}_{13}\left[\frac{(1+\bar{w}_1)\beta^2}{\bar{w}_1\bar{C}^1}\right]$$

$$+ \bar{e}_{23}\left[-(1-\alpha)\frac{(\bar{w}_1-\bar{w}_2)+(1-\beta)(1+\bar{w}_2)}{\bar{w}_1}\right] - \bar{e}_{34}\left[\frac{\bar{w}_1-\bar{w}_2}{\bar{w}_1}\right],$$

$$B_3 = -(1-\alpha)\bar{C}^1_{ww} + \bar{e}_{14}\left[\frac{-\alpha^2(1-\beta)}{\bar{C}^1}\right] + \bar{e}_{13}\left[\frac{(1-\alpha)\beta^2}{\bar{w}_1\bar{C}^1}\right] + \bar{e}_{23}\left[\frac{\beta(1-\alpha)\bar{C}^2}{\bar{w}_1\bar{C}^1}\right]$$

$$+ \bar{e}_{34}\left[\frac{-\beta(1-\beta)}{\bar{w}_1\bar{C}^1}\left(\frac{\alpha\bar{w}_1}{\beta} - \frac{(1-\alpha)\bar{w}_2}{1-\beta}\right)\right].$$

III.13 $$\frac{d\bar{w}_{av}}{\bar{w}_{av}} = \frac{dI}{N}\left\{\frac{F_1(k-1) + F_2(a-\alpha) + F_3\Delta Q_1}{\bar{w}_{av}(1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)\cdot|A|}\right\}, \text{ where}$$

$$F_1 = \left\{-\alpha^2\bar{C}^2_{ww}-(1-\alpha)^2\bar{C}^1_{ww}\right\} + \bar{e}_{12}\left[\alpha^2(1-\alpha)^2\frac{(1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)^2}{\bar{C}^1\cdot\bar{C}^2}\right]$$

$$+\left\{\bar{e}_{23}\left[(1-\alpha)^2\frac{(\bar{C}^2+\alpha\bar{w}_1)^2}{\bar{w}_1\bar{C}^2}\right] + \bar{e}_{14}\left[\alpha^2\frac{(\bar{C}^1+(1-\alpha)\bar{w}_2)^2}{\bar{w}_2\bar{C}^1}\right]\right\}$$

continued.

$$+ \bar{e}_{34}\left[\frac{(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)^2}{\bar{w}_1\bar{w}_2}\right] + \left\{\bar{e}_{24}\left[\frac{\alpha^2(1-\beta)^2}{\bar{w}_2\bar{C}^2}\right] + \bar{e}_{13}\left[\frac{(1-\alpha)^2\beta^2}{\bar{w}_1\bar{C}^1}\right]\right\},$$

$$F_2 = \left\{\alpha(1+w_2)\bar{C}^2_{ww} - (1-\alpha)(1+\bar{w}_1)\bar{C}^1_{ww}\right\} - \left\{\bar{e}_{12}\left[\alpha(1-\alpha)\frac{(1+\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)^2(\beta-\alpha)}{\bar{C}^1\cdot\bar{C}^2}\right]\right\}$$

$$+ \left\{\bar{e}_{23}\left[\frac{(1-\beta)(1-\alpha)(\bar{C}^2+\alpha\bar{w}_1)(1+\bar{w}_2)}{\bar{w}_1\bar{C}^2} + \frac{(\bar{w}_1-\bar{w}_2)(1-\alpha)}{\bar{w}_1\bar{C}^2}\left((1-\beta)\alpha\bar{w}_1-(1-\alpha)(\bar{C}^2+\alpha\bar{w}_1)\right)\right]\right.$$

$$\left.- \bar{e}_{14}\left[\frac{\alpha\beta}{\bar{w}_2\bar{C}^1}(\bar{C}^1+(1-\alpha)\bar{w}_2)(1+\bar{w}_2) + \frac{(\bar{w}_1-\bar{w}_2)}{\bar{w}_2\bar{C}_1}\alpha\left(\beta(1-\alpha)\bar{w}_2 + \alpha(\bar{C}^1+(1-\alpha)\bar{w}_2\right)\right]\right\}$$

$$+ \left\{\bar{e}_{13}\left[\frac{(1-\alpha)(\beta^2)(1+\bar{w}_1)}{\bar{w}_1\bar{C}^1}\right] - \bar{e}_{24}\left[\frac{\alpha(1-\beta)^2(1+\bar{w}_2)}{\bar{w}_2\bar{C}^2}\right]\right\}$$

$$- \left\{\bar{e}_{34}\left[\frac{(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2)(\bar{w}_1-\bar{w}_2)}{\bar{w}_1\bar{w}_2}\right]\right\},$$

$$F_3 = \left\{-\alpha^2\bar{C}^2_{ww} + (1-\alpha)^2\frac{\bar{C}^1_{ww}\bar{C}^1}{\bar{C}^2}\right\} + \left\{\bar{e}_{34}\left[\beta(1-\beta)\frac{(\alpha\bar{w}_1+(1-\alpha)\bar{w}^2)}{\bar{w}_1\bar{w}_2\bar{C}^2}\left(\frac{\alpha\bar{w}_1}{\beta} - \frac{(1-\alpha)\bar{w}_2}{1-\beta}\right)\right]\right\}$$

$$+ \left\{\bar{e}_{14}\left[\frac{\alpha^2(1-\beta)}{\bar{w}_2\bar{C}^2}(\bar{C}^1+(1-\alpha)\bar{w}_2)\right] - \bar{e}_{23}\left[\frac{\beta(1-\alpha)^2}{\bar{w}_1\bar{C}^1\bar{C}^2}(\bar{C}^{2^2}+\alpha\bar{w}_1\bar{C}^1)\right]\right\}$$

$$+ \left\{\bar{e}_{24}\left[\frac{\alpha^2(1-\beta)^2}{\bar{w}_2\bar{C}^2}\right] - \bar{e}_{13}\left[\frac{(1-\alpha)^2\beta^2}{\bar{w}_1\bar{C}^2}\right]\right\}.$$

The change in the wage rates is expressed as a sum of three terms: the first reflecting the effect of the difference between the capital holdings of the immigrant and indigenous workers; the second, the effect of the difference between the intersectoral distributions of immigrant labour and the indigenous work force; and the third, the effect of the different consumption patterns of the immigrant.

These results are what we would expect in that more immigrant capital yields higher wage rates, i.e., the coefficients A_1, B_1 and F_1 are unambiguously positive in sign. If $a = \alpha$ and $\Delta Q_1 = 0$, the one sector model of section I. suffices. Product substitution and the endogenous labour supply decision may be taken account of in a numerical analysis by using larger elasticities of factor substitution in equation I.8.[21/] However, the signs of the other coefficients are ambiguous. Thus, if the immigrant population is not a microcosm of the indigenous population, a large degree of asymmetry in the economy, as indicated, for example, by large nonzero values for some of the terms in curly brackets in the expressions for F_2 and F_3, might yield qualitatively different results from those described in I. We now consider briefly a few such "unexpected" possibilities.

Suppose that the elasticity of substitution τ_{12} between the goods y_1 and y_2 is much greater then the other elasticities of substitution in consumption, so that we may ignore for the moment all but the first two terms in the expressions for A_2, B_2 and F_2. If sector 1 is capital intensive, i.e. if $\beta > \alpha$, then the sign of A_2 is unambiguously negative, and the effect of a disproportionate influx of immigrant labour into sector 1, i.e. $a-\alpha > 0$, will be to depress the wage rate in that sector. However, if sector 1 is labour intensive, and if τ_{12} is sufficiently larger than σ_1 and σ_2, it is possible that $a > \alpha$, $k < 1$, and $\Delta Q_1 = 0$, and yet that $dw_1 > 0$ and $dw_2 > 0$; that is, a disproportionate influx of immigrant labour, possessing relatively little capital, into the labour intensive sector, may result in an increase in both wage rates.

This somewhat paradoxical result is similar to that derived by Diewert in [11] where he showed that it is conceivable that an easing of restrictions to entry into a unionized industry could lead to higher wages for union members. In an attempt to rationalize our result, we may adopt the scenario he used as follows: A disproportionate influx of immigrants into labour intensive sector 1 leads employers there to switch to more labour intensive techniques. At the same time they expand their scale of output, and lure both capital and labour away from sector 2, the latter by raising w_1 slightly. Under appropriate conditions this can create a relative scarcity of labour in sector 2, and hence w_2 rises.[22/] This may push up the price of y_2 sufficiently so that the demand for the good falls, and the "optomistic" behaviour on the part of producers in sector 1 is validated by the market.

As another example, suppose that the dominant elasticity of substitution is that existing between the two leisure goods, i.e., suppose that there is a very large degree of substitution between the labour offered to each sector in response to changes in the wage rates in each sector. If there is a large influx of immigrant labour into the sector having the lower wage rate, i.e. if say $a < \alpha$ and $w_1 > w_2$, and if τ_{34} is much larger than the other elasticities, it is conceivable that both wage rates increase, even though $k < 1$ and $\Delta Q_1 = 0$. This might be explained as follows: A disproportionate influx of immigrant labour into the low wage sector 2 and the influx of immigrant capital, (which may be thought of as instantaneously distributed between the sectors in the same proportion as the indigenous capital), may encourage employers in sector 1 to raise the wage rate, but less than might be indicated by the initial change in the labour capital ratio in the sector. Producers in sector 2

adopt more labour intensive techniques but also attract some capital from sector 1, thus validating the previous behaviour of employers there. Under appropriate conditions, the net effect may be to create a relative scarcity of labour in sector 2, thus causing w_2 to rise.[23/]

In our discussion till now we have concentrated on the effect of the different intersectoral distribution of immigrant labour. If we now allow immigrants to have different preferences, a not unreasonable assumption at least for their first few years of residence, the direction of the effect which this shift in consumption will have on the average wage is determined by the sign of F_3. If only elasticities of factor substitution are significant, F_3 has the same sign as the difference

$$\frac{\alpha\bar{w}_1}{\beta\sigma_1} - \frac{(1-\alpha)\bar{w}_2}{(1-\beta)\sigma_2} = \frac{1}{\sigma_1}\left[\frac{\text{wage bill}}{\text{capital bill}}\right]_{\text{sector 1}} - \frac{1}{\sigma_2}\left[\frac{\text{wage bill}}{\text{capital bill}}\right]_{\text{sector 2}}.$$

Even if $\left[\frac{\text{wage bill}}{\text{capital bill}}\right]_{\text{sector 1}} = \left[\frac{\text{wage bill}}{\text{capital bill}}\right]_{\text{sector 2}}$, as might be the case if we thought of sector 1 as consisting of the construction and real estate industries and of sector 2 as encompassing the rest of the economy, (in which case $\Delta Q_1 > 0$ is plausible), it is still possible that the elasticities of substitution are such that the net effect is to raise w_{av}, i.e., if $\sigma_2 > \sigma_1$, which is also quite plausible, the net effect on wages need not be repressive. However, if the dominant elasticity is the elasticity of substitution between two leisure goods, then F_3 has the same sign as $\left[\frac{\text{wage bill}}{\text{capital bill}}\right]_{\text{sector 1}} - \left[\frac{\text{wage bill}}{\text{capital bill}}\right]_{\text{sector 2}}$.

Of course it is also possible that in equation III.13, the terms involving $(a-\alpha)$ and ΔQ_1 may act to reinforce a shortage of

immigrant capital, thus resulting in a **larger** drop in wages then would have been indicated by a one sector model. Numerically, this might be captured in the one sector model through use of a smaller elasticity of capital-labour substitution.

Formulae for the changes in real wages for workers in each sector, in per capita real income and in labour's share of the national product may now be worked out precisely as in the previous sections. Consider briefly an "average" worker with no capital. His real wage, or indirect utility function u, is given by:

II.14
$$u = \frac{w_1H_1 + w_2H_2}{e[C^1(1,w_1),C^2(1,w_2),w_1,w_2]} .$$

We find that:

II.15
$$\frac{du}{\bar{u}} = \frac{H_1dw_1 + H_2dw_2}{(\bar{w}_1H_1 + \bar{w}_2H_2)\cdot\bar{e}} = \frac{\frac{H_2}{1-\alpha}\,dw_{av} + \alpha\left(\frac{H_1}{\alpha} - \frac{H_2}{1-\alpha}\right)dw_1}{(\bar{w}_1H_1 + \bar{w}_2H_2)\cdot\bar{e}} .$$

If initially consumer-workers are supplying equal amounts of labour to each sector relative to the maximum amounts they are capable of supplying, i.e., if $\frac{H_1}{\alpha} = \frac{H_2}{1-\alpha}$, then the "average" worker will benefit in real terms only if the average wage rises. Otherwise, a fall in the average money wage is not necessarily inconsistent with a rise in the average real wage; for example, if initially more labour is supplied to sector 2 than to sector 1 relative to the maximum potential supplies, and if w_1 increases.

<u>Numerical Example</u>

We divide the economy into two sectors corresponding roughly to a union and nonunion sector, as done by Diewert in [11]. Sector 1 is defined to consist of the following industries: construction,

manufacturing, transportation, public utilities, communications, government enterprises, mining and forestry. Sector 2 consists of agriculture, trade, finance, real estate, and services. We use Diewert's estimates that $\alpha = \frac{1}{2}$, $\beta = \frac{1}{3}$.

Once again we consider the 1969-70 time period, estimate $k = .17$ and $\alpha\bar{w}_1+(1-\alpha)\bar{w}_2 = 4$. From [7] we estimate that:

$$\frac{\alpha\bar{w}_1}{\alpha\bar{w}_1+(1-\alpha)\bar{w}_2} = \frac{\text{wage bill in sector}}{\text{total wage bill}} = .56, \quad \frac{(1-\alpha)\bar{w}_2}{\alpha\bar{w}_1+(1-\alpha)\bar{w}_2} = \frac{\text{wage bill in sector 2}}{\text{total wage bill}} = .44.$$

We deduce that $\bar{w}_1 = 4.5$ and $\bar{w}_2 = 3.5$.

It was more difficult to estimate the proportion of immigrant labour going into each sector. Using statistics in the intended occupations of immigrants [5] and the occupation-industry matrix from the Labour Force Survey, April 1969, we find that "a" and α did not differ significantly. Hence, we proceed on the assumption that $a = \alpha$.

Rather than postulating various combinations of values for all eight elasticities of substitution involved in the model, and computing the induced change in the wage rates for each such combination, we reduce our workload by ignoring all but one of the elasticities of substitution in consumption - τ_{12}, that between the two production goods. The other elasticities are probably small enough so that, with the existing degree of asymmetry between the two sectors, this simplification will not change our results by very much. (For example, union restrictions probably make τ_{34} quite small.) Noting that the purpose of this numerical analysis is purely illustrative, we proceed, with total peace of mind, to present our simplified numerical example.

Using equations III.12 and III.13, together with the figures given above, the equations for the induced change in the wage rates due to immigration become:

$$\frac{dw_1}{\bar{w}_1} = \frac{dI}{N}\left\{\frac{\frac{1}{6}\sigma_2[-.83+\Delta Q_1]-.37\tau_{12}}{D}\right\},$$

III.14
$$\frac{dw_2}{\bar{w}_2} = \frac{dI}{N}\left\{\frac{\frac{1}{12}\sigma_1[-.83-\Delta Q_1]-.46\tau_{12}}{D}\right\},$$

$$\frac{dw_{av}}{\bar{w}_{av}} = \frac{dI}{N}\left\{\frac{-[.76\sigma_2-.31\sigma_1-3.65\tau_{12}]+\Delta y_1[.92\sigma_2-.37\sigma_1]}{10 \times D}\right\}, \quad \text{where}$$

$$D = .089\sigma_1\sigma_2 + .375\sigma_2\tau_{12} + .146\sigma_1\sigma_{12}.$$

The role of ΔQ_1 may be clarified by the following table:

TABLE III.1

ΔQ_1	Corresponding percent of immigrant income spent on good 1 (approximate figures)
-4.2	0%
-1	40%
0	50%
1	60%
2	70%
4.2	100%

The indigenous worker, of course, spends approximately equal portions of his income on the two goods. (Note that $\bar{C}^1 = \bar{C}^2 = 2.5$). With the present definition of the two sectors it is difficult to say what a reasonable value for ΔQ_1 would be. In our calculations below, we try,

in turn, $\Delta Q_1 = -1, 0, +1$.

The results of our calculations, for different postulated values for σ_1, σ_2 and τ_{12}, and under the assumption that $\frac{dI}{N} = .95\%$, are shown in TABLE III.2. The figures indicate that the average wage will fall, though the wage rates in one of the sectors may rise slightly. Corresponding to the various assumptions we have made regarding the consumption pattern of immigrants, they define a much larger range of values for the change in wages than did the figures in Section I., though a smaller range than indicated in TABLE II.1. Indeed, if we were to assume that $\Delta Q_1 = 2$, then an appropriate combination of values for the elasticities would lead us to conclude that the average wage had been induced to rise. This emphasizes, once again, the fact that, in general, and even in the Canadian context, a one sector model may not enable us to draw correct qualitative, not to mention, quantitative, conclusions regarding the effects of immigration on wages. In the Canadian context, a different definition of the sectors, corresponding more closely to the immigrant's unique consumption pattern in the first few years, and to their (relatively) nonuniform (?) dispersion among the various industries, would perhaps alter significantly the estimates of $\frac{dw_{av}}{\bar{w}_{av}}$.

TABLE III.2 - INDUCED CHANGES IN THE WAGE RATES IN THE "UNION" AND "NONUNION" SECTORS

	τ_{12}*	σ_1**	σ_2***	D	$\frac{dw_1}{\bar{w}_1}$	$\frac{dw_2}{\bar{w}_2}$	$\frac{dw_{av}}{\bar{w}_{av}}$
$\Delta Q_1 = -1$	0	1/2	1/2	.022	-6.5%	.22%	-3.5%
	0	1/2	3/2	.066	-6.5%	.07%	-3.6%
	0	3/2	1/2	.066	-2.2%	.22%	-1.3%
	0	3/2	3/2	.198	-2.2%	.07%	-1.4%
	1	1/2	1/2	.280	-1.8%	-1.6%	-1.7%
	1	1/2	3/2	.701	-1.1%	-.62%	-.89%
	1	3/2	1/2	.470	-1.1%	-.90%	-1.0%
	1	3/2	3/2	.847	-.92%	-.50%	-.73%
$\Delta Q_1 = 0$	0	1/2	1/2	.022	-3.0%	-1.6%	-2.4%
	0	1/2	3/2	.066	-3.0%	-.53%	-1.9%
	0	3/2	1/2	.066	-1.0%	-1.6%	-1.3%
	0	3/2	3/2	.198	-1.0%	-.53%	-.79%
	1	1/2	1/2	.280	-1.5%	-1.7%	-1.6%
	1	1/2	3/2	.701	-.79%	-.67%	-.74%
	1	3/2	1/2	.470	-.89%	-1.1%	-.98%
	1	3/2	3/2	.847	-.65%	-.63%	-.64%
$\Delta Q_1 = 1$	0	1/2	1/2	.022	.68%	-3.2%	-1.0%
	0	1/2	3/2	.066	.68%	-1.1%	-.10%
	0	3/2	1/2	.066	.23%	-3.2%	-1.3%
	0	3/2	3/2	.198	.22%	-1.1%	-.36%
	1	1/2	1/2	.280	-1.2%	-1.8%	-1.5%
	1	1/2	3/2	.701	-.44%	-.72%	-.57%
	1	3/2	1/2	.470	-.71%	-1.4%	-1.0%
	1	3/2	3/2	.847	-.37%	-.77%	-.55%

*τ_{12} is the elasticity of substitution in consumption between "union" and "nonunion" goods.

**σ_1 is the capital-labour elasticity of substitution in the "union" sector.

***σ_2 is the capital-labour elasticity of substitution in the "nonunion" sector.

FOOTNOTES

1. I am indebted to Ken Scott of the Research Branch, Department of Manpower and Immigration, and to Erwin Diewert of the University of British Columbia, for suggesting the topic to me and for some helpful comments.

2. This qualitative effect on the distribution of income is assumed throughout the literature. Mishan and Needleman, however, are the only ones to develop a mathematical model in order to "derive" and quantify the result. The term "regressive" is used by them to describe a change in factor prices which see wages decline relative to the rental rate of capital.

3. Most research in Canada into the economics of immigration has been concerned primarily with: explaining changes in migration flows; examining the economic (and social) adjustment patterns of immigrants, and estimating the monetary value of human capital brought into Canada by immigrants. Only recently is more attention being paid to the impact which immigration has on the welfare of Canadians. It is hoped that the models developed in this paper can contribute something to an analysis of this impact.

4. The assumption of constant returns to scale in production may be dropped easily. If we assume that profits, like income from labour and capital, are spent on consumption of y, then the arguments given below may be adjusted to allow for variable nonincreasing returns to scale. Since available data do not permit reasonable estimates of the degree of nonhomogeneity of production in the Canadian economy, and since it was felt that the number of equations in the paper had already exceeded the optimum value, we do not deal with variable returns to scale.

5. If the underlying production function is linear homogeneous, the we have $C(y;p_1,\ldots,p_N) = C(1;p_1,\ldots,p_N)\cdot Y$, where $C(1;p_1,\ldots,p_N)$ is the unit cost function.

6. For a derivation of this formula see [9].

7. If immigrants and emigrants have similar characteristics, "I" could refer to the net number of immigrants, and the model would describe the effects of net immigration. Otherwise, the effects of immigration alone, and of emigration alone, may be determined separately, (the latter by considering $-I$ rather than I), and the effects of net immigration derived as their sum.

8. All empirical studies with which we are familar have found that $\sigma > 0$, and so we assume this to be the case throughout our discussion.

9. In the case of a nonhomothetic production function, i.e., if there is a bias towards one of the factors upon expansion, this is no longer true. Rather wages will increase or decrease according as $k-a\cdot\gamma > 0$ or $k-a\cdot\gamma < 0$, where γ is a positive constant with magnitude greater than 1 or less than 1 according as there is a bias towards capital or labour.

10. If we assume that a rise in relative wages will cause the consumer-worker to consume more of y , i.e., if the elasticity of substitution in consumption between y and leisure is greater than zero, then the effect of an elastic supply of labour would be to increase the "effective" elasticity in equation I.8. A corresponding statement may be made on the effect of an elastic supply of capital.

11. Actually, if $C_{ww}(1,\bar{w}) \lneqq 0$, or equivalently if $\sigma \gneqq 0$, then u will actually increase with a small influx of immigrants. This is a second order change, however, which we ignore.

12. We could, of course, consider a consumer-worker having any amount of capital, in the range 0 to N . We would find that those with a smaller than average share of capital (1 unit) benefit in real terms only if wages rise relative to rental rates, while those with a larger than average share benefit only if wages fall. If we have decreasing returns to scale in production, diseconomies of scale may alter these conclusions.

13. We have no idea how close "declared" savings are to "actual" savings, nor are we certain that all immigrant savings are invested in physical capital. However, for illustrative purposes our estimate of k suffices.

14. From [7] we estimate labour income for 1969 as the sum of wages, salaries and supplementary labour income (43,203) and military pay and allowances (898), and capital income for 1969 as the sume of net income received by farm operators from farm production (1,644), net income of non-farm unincorporated business including rent (4,410), and interest, dividents and miscellaneous investment income (4,961).

15. The comments we made in I. regarding variable returns to scale also apply in this section.

16. Our analysis assumes that native and immigrant labour will experience the same rate of unemployment. Thus, N is the size of the total labour force and α and $1-\alpha$ are the proportions of the total labour force in each occupational group.

17. See footnote 14.

18. If the utility function were homogeneous of degree $\alpha > 0$, it could be transformed into a linear homogeneous function which would equivalently represent the consumer worker's preferences.

19. Our consumer-workers are assumed to be solving the following utility maximization problem:

$$\max_{y_1,y_2,L_1,L_2} U_1(y_1,y_2,L_1,L_2)$$

subject to the condition that:

$$y_1,y_2 \geq 0$$

$$L_1,L_2 \geq 0$$

$$L_1 \leq H_1\ ,\ L_2 \leq H_2 \quad \text{(time constraint)}$$

$$p_1y_1+p_2y_2 \leq 1+w_1L_1+w_2L_2 \quad \text{(budget constraint)},$$

where L_1 and L_2 are the amounts of labour supplied to sectors 1 and 2, and where U_1 satisfies, with respect to the variables $y_1,y_2,-L_1$ and $-L_2$, the conditions quoted above for U. Using the fact that $Le_i = H_i-L_i$, $i = 1,2$, the above problem is easily seen to transform to the new problem:

$$\max_{y_1,y_2,Le_1,Le_2} U(y_1,y_2,Le_1,Le_2)$$

subject to the conditions that: $y_1,y_2,Le_1,Le_2 \geq 0$

$$Le_i \leq H_i \quad i = 1,2 \quad \text{(time constraint)}$$

$$p_1y_1+p_2y_2 + w_1Le_1+w_2Le_2 \leq 1+w_1H_1+w_2H_2$$

(budget constraint).

This the form in which the problem of utility maximization is considered in the text. We see that the leisure goods have prices equal to the wage rates, and that a generalized full income may be defined to be equal to $1+w_1H_1+w_2H_2$.

20. The supply of immigrant labour is assumed to be inelastic and so the leisure "goods" do not enter into the immigrants utility function. An elastic supply would not change our first order approximations to the induced changes in wages.

21. This was noted by Mishan and Needleman in their paper. However, they did not consider the effects of deleting these assumptions.

22. A necessary condition in order that w_2 be induced to rise is that $(k-1)(1-\alpha)+(a-\alpha)(\alpha-\beta) > 0$. Similarly, w_1 will rise only if $(k-1)\alpha+(a-\alpha)(\alpha-\beta) > 0$.

23. This can occur only if we have $(\bar{w}_1-\bar{w}_2)(\alpha-a)+(k-1)(\alpha\bar{w}_1+(1-\alpha)\bar{w}_2) > 0$.

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